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THESIS

SIMULATING THE EFFECTIVENESS OF AN ALTERNATIVE SALARY AUCTION MECHANISM

by

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December 2006

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      This research simulates the effectiveness of an alternative auction mechanism for Assignment Incentive Pay (AIP) that has potential for reducing the U.S. Department of Defense's (DoD's) cost. A recent student thesis studying the application of salary auctions and matching in an assignment setting determined that there are two major complications in an assignment auction which affect the incentive of bidders to submit a truthful valuation of the jobs. An alternative auction mechanism that combined elements of both auction theory and matching was proposed to overcome these complications. This study further defines this alternative auction mechanism and presents a simulation setup for testing the effectiveness of the mechanism. Simulation is carried out and the mechanism is evaluated based on defined operational performance and efficiency measures. The objective of this thesis is to evaluate the benefits of the alternative auction mechanism to DoD.

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**SIMULATING THE EFFECTIVENESS OF AN
ALTERNATIVE SALARY AUCTION MECHANISM**

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ABSTRACT

This research simulates the effectiveness of an alternative auction mechanism for Assignment Incentive Pay (AIP) that has potential for reducing the U.S. Department of Defense's (DoD's) cost. A recent student thesis studying the application of salary auctions and matching in an assignment setting determined that there are two major complications in an assignment auction which affect the incentive of bidders to submit a truthful valuation of the jobs. An alternative auction mechanism that combined elements of both auction theory and matching was proposed to overcome these complications. This study further defines this alternative auction mechanism and presents a simulation setup for testing the effectiveness of the mechanism. Simulation is carried out and the mechanism is evaluated based on defined operational performance and efficiency measures. The objective of this thesis is to evaluate the benefits of the alternative auction mechanism to DoD.

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TABLE OF CONTENTS

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LIST OF TABLES

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                                                                    Psalm 106: 1

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I.   INTRODUCTION

A.   BACKGROUND

The U.S. Navy has introduced the Assignment Incentive Pay (AIP) Program to deal with recurrent manning shortages in certain billets. Under the AIP Program, selected sailors receive the monthly bonus pay they requested for the duration of their tours in hard-to-fill, AIP-eligible billets. The AIP Program has been implemented using an auction or bidding system, which is expected to be a cost-effective means of getting sailors to voluntarily accept assignment to billets that are traditionally less desirable. AIP rates can vary for individual billets and depend on sailors' bidding behavior, which in turn is affected by sailors' preferences and alternatives as well as the structure of the bidding system and its implementation.

A recent student thesis studying the application of salary auctions and matching in an assignment setting determined that there are two major complications in an assignment auction that affect the incentive of bidders to submit a truthful valuation for the jobs. Firstly, because each bidder can only win one post, bidders may have an incentive to overstate their willingness-to-accept or reservation wage if they anticipate that the system must allocate them a post, particularly if the number of open posts is at or above the number of bidders. Secondly, standard auction formats assume one-sided preferences in which bidders care about the items (in this case, assignments) for which they are bidding while the bid-taker (in this case, the military) has no preference over which bidders win the auctions. If a sailor quality variable (reflecting the military's assessment of the value that a particular sailor or type of sailor will bring to a particular assignment) is somehow combined with the dollar amount of each sailor's bid to determine the winner of the assignment auctions, however, high quality bidders have an incentive to maximize their personal surplus by submitting a higher bid than their true minimum willingness-to-accept for the assignment.

An alternative auction mechanism that combined elements of both auction theory and matching was proposed to overcome these complications and potentially reduce the

U.S. Department of Defense's (DoD's) cost. While the optimality of this proposed mechanism has been proven theoretically, it has not been tested to see if it works in reality.

B.   PURPOSE

This study further defines the proposed alternative auction mechanism and presents a simulation setup for testing the effectiveness of the mechanism. The mechanism will be evaluated based on defined operational performance and efficiency measures.

The following research questions are addressed:

- Does the alternative auction mechanism work?

    o In particular, does the mechanism properly assign sailors to billets consistently and easily?

    o Does the mechanism also determine the incentive pay amounts associated with each assignment consistently and easily?

- Is the alternative auction mechanism efficient?

- What are the benefits of the alternative auction mechanism to DoD?

C.   SCOPE AND LIMITATIONS

1.   Scope

The scope of this thesis includes:

(1) a review of salary auctions for assignments in the military,

(2) definition of the alternative auction mechanism to be tested,

(3) simulation setup,

(4) carrying out simulations to test the effectiveness of the mechanism, and

(5) an evaluation of the operational performance and efficiency of the mechanism.

The thesis will conclude with the benefits of the alternative auction mechanism to DoD.

2. Limitations

This research is not an empirical study and does not determine whether (and by how much) submitted bids will vary from a sailor's true willingness-to-accept for a particular assignment. It is also outside the scope of this research to consider the possibility of ex-post bargaining or intervention. The testing of the mechanism is based on one scenario involving an equal number of billets and sailors as well as the same valuation ranges. Extended testing with alternative settings by varying the number of sailors and billets as well as the valuation ranges is a recommended follow-up from this research.

D. EXPECTED BENEFITS OF THE STUDY

This study will help improve knowledge and understanding of the interaction between auction theory and matching mechanisms as applied to assignments. It will also help to determine if there is a more effective auction mechanism for Assignment Incentive Pay (AIP).

E. ORGANIZATION OF THE THESIS

An overview of auction theory is covered in Chapter II. Chapter III explains the application of auctions to assignments in the military. The simulation setup and evaluation measures are presented in Chapter IV. Chapter V reports the simulation results and provides an evaluation of the alternative auction mechanism. Chapter VI concludes this study with a summary, conclusions and recommendations for future research.

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II. AUCTION THEORY

A. BACKGROUND

The root word for auction is "auctio" meaning "increase". According to the Merriam-Webster Online Dictionary, an auction is "a sale of property to the highest bidder"[1]. An economic definition will be that an auction is "a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participants"[2].

An auction is essentially a method for transacting a commodity or resource that has an undetermined or variable price. The price is determined through competition among buyers or sellers who bid for the right to buy or sell, with the item allocated to the bidder with the best offer. Auctions can be categorized as single-object or multi-object auctions. The role of information or type of valuation determines if an auction is a common value or independent private-values auction. Auctions also differ in terms of rules specifying the roles of buyers and sellers; open as opposed to sealed bids; first-price as opposed to second-price payments; and the level of security or privacy among others.

Auctions have existed for centuries, dating as far back as the Babylonian and Roman empires,[3] and have been used to transact almost anything, ranging from antiques to environmental licenses. Among the most famous auction houses in the world are Christie's, Sotheby's and Bonhams. Auctions have also moved online with companies such as eBay operating global trading platforms where individuals and businesses can trade a wide variety of products and services.

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[1] "Merriam-Webster Online," http://www.m-w.com/cgi-bin/dictionary (assessed October 2, 2006).

[2] R. Preston McAfee and John McMillan, "Auctions and Bidding," Journal of Economic Literature Vol. XXV (June 1987), 701.

[3] Paul Klemperer, Auctions: Theory and Practice (Princeton, NJ, USA: Princeton University Press, 2004), 15.

B. KEY FEATURES

Some key features of auctions are given below:

1. Common Value versus Independent Private-Values Auctions

a. *Common Value Auctions*

In common value auctions, the value of the auction item is the same for each participant; however, different participants may have different information about the potential value of the item. As an example, participants auctioning for a jar full of quarters may not have complete information about the true value of the jar, but the jar will be worth the same amount to each participant. Asymmetric information in this case leads to different valuations and gives rise to the winner's curse, in which auction winners "curse" their win when they realize that they won because they overestimated the item's value, which led them to overbid and suffer a loss.

b. *Independent Private-Values Auctions*

In independent private-values auctions, each bidder knows his/her personal value for the item, but he/she is uncertain about other bidders' values, which differ based on taste and preference. Each bidder's valuation is independent of and unaffected by any information about others' valuations, unlike the common value auction in which a bidder is likely to revise his value estimate based on other bidders' values. This does not, however, preclude private-values auction bidders from changing their bids to gain a strategic advantage if they receive information about other bidders' values.

2. First-price versus Second-price Auctions

a. *First-price Auctions*

In first-price auctions, the highest (lowest) bidder pays (receives) the winning bid they submitted.

b. *Second-price Auctions*

In a second-price auction, the highest (lowest) bidder pays (receives) the second highest (lowest) bid rather than the one they submitted, or the first excluded bid in a multi-object auction.

3. Forward versus Reverse Auctions

a. *Forward Auctions*

In a forward auction, an item is put up for sale by the seller, who is the bid-taker. Multiple potential buyers are the bidders who put up increasing bids for the item. The highest bidder wins the right to buy the item at a price determined when the bidding ends, which can be the winning bid or the second highest bid. Forward auctions have been used to sell almost anything, from wives and slaves in early auctions, to manuscripts, antiques, agricultural produce and even corporations more currently.

b. *Reverse Auctions*

A reverse auction conversely has one buyer who is the bid-taker and many sellers who are the bidders submitting decreasing bids. The right to sell is won by the bidder with the lowest bid and the sale price can be the winning bid or the second lowest bid. Reverse auctions have been used by procurement agencies to secure supplies of a requested good or service from the supplier with the lowest bid to accept the contract.

4. Open versus Sealed-bid Auctions

a. *Open Auctions*

An open auction is one that is conducted such that "bidders are able to observe their rival's bids and accordingly, if they choose, revise their own bids"[4]. Bids can either be announced by an auctioneer, called out by the bidders themselves or submitted and posted electronically. Open forward auctions have been used to sell art pieces in which several potential buyers bid against one another, escalating the price until an eventual winner emerges with the highest bid.

b. *Sealed-bid Auctions*

In a sealed-bid auction, bidders each simultaneously submit only one sealed bid. Sealed-bid auctions have been used to sell mineral rights, artwork and real estate.

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[4] R. Preston McAfee and John McMillan, "Auctions and Bidding," Journal of Economic Literature Vol. XXV (June 1987), 702.

5.   Single versus Multi-Object Auctions

a.   *Single-Object Auctions*

In a single-object auction, a single item is auctioned at any one time. The simple single-object auction has been the focus of theoretical research for many years and it has been researched extensively.

b.   *Multi-Object Auctions*

A multi-object auction involves auctioning several items at one time. Multi-object auctions have been commonly analyzed as if they were a number of independent single-object auctions. However, treating multi-object auctions as simultaneous single-object auctions is inappropriate when the value of the item acquired depends on the other items the bidder acquires or does not acquire because the other items are either complements or substitutes. The approximation may also be inappropriate "if bidders have budget restrictions, capacity constraints, or, in general, have non-linear utility functions"[5].

C.   TYPES OF AUCTIONS

There are four basic types of auctions in use: the English auction, the Dutch auction, the first-price sealed-bid auction, and the second-price sealed-bid auction. Each of the four types of auctions will be described and analyzed in the context of a forward auction; however the reverse auction equivalent should be easily understood to be the mirror image of what will be described.

1.   English Auction

The English auction is also known as the oral, open or ascending-bid auction and is the most commonly used auction form. Bidders compete openly against each another, successively raising the price until no one is willing to bid further. The highest bidder then pays the price that is equivalent to the second highest bid plus the bid increment. Antiques and artwork are frequently sold through English auctions.

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5 Richard Engelbrecht-Wiggans and Robert J. Weber, "An Example of a Multi-Object Auction Game," Management Science Vol. 25 No. 12 (December 1979), 1272.

Assuming private-values, the equilibrium strategy of bidders in a forward English auction is to bid up to his valuation or maximum willingness-to-pay, WTP, but no further regardless of what others bid. If a participant's final bid is lower than his maximum WTP, he foregoes potential economic rent, which is defined as his valuation minus price paid. If his final bid is higher than his maximum WTP, he will suffer a loss. The winner will be the bidder with the highest valuation who effectively pays the second highest valuation if the bid increment is negligible. This auction form is thereby efficient with the item allocated to the bidder who values it most and truth-revealing because the dominant bidding strategy dictates that all bidders, with the exception of the winner, reveal their true valuation.

2.   Dutch Auction

In the Dutch auction, also known as the descending-bid auction, the converse occurs, with the auctioneer or some sort of electronic device, such as a clock, calling a high starting price that is lowered until one bidder accepts and pays the going price. The Dutch auction has been used to sell tulips in the Netherlands, fish in Israel and tobacco in Canada.

Under this auction form, there is no dominant bidding strategy. Each participant decides on his best bid based on his valuation and expectation of his competitors' valuation and bidding strategies. Nevertheless, it can be proven mathematically that at a Nash equilibrium, the decision rule for each bidder in a forward auction is to bid below his true valuation. The Dutch auction is therefore not truth-revealing and unless all bidders have the same information (or expectations), "there is no assurance that the equilibrium outcome will be efficient"[6], unlike the English auction that always results in a Pareto-efficient allocation.

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[6] Paul Milgrom, "Auctions and Bidding: A Primer" Journal of Economic Perspectives Vol. 3 No.3 (Summer 1989), 9.

3. First-price Sealed-bid Auction

Under the first-price sealed-bid auction, all participants submit bids simultaneously such that no one knows the bid put in by others. In a forward auction, the highest bidder wins and pays the price they submitted.

The first-price sealed-bid auction is analogous to the Dutch auction because each participant faces the same situation; they must decide how much to bid not knowing how others are going to bid and pay their own bid if they win. The first-price sealed-bid auction is therefore not truth-revealing and can result in an inefficient outcome unless all bidders have the same information (or expectations).

4. Second-price Sealed-bid Auction

The second-price sealed-bid auction is also known as the Vickrey auction and is similar to the first-price sealed-bid auction except that in a forward auction, the highest bidder wins and pays the second highest bid rather than the one they submitted. The winner therefore pays a price that is determined by others' bids and not his bidding actions. This being the case, a bidder's dominant strategy is to "submit a bid equal to his true reservation price, for he then accepts all offers that are below his reservation price and none that are above"[7]. Each bidder bids his true valuation regardless of what others bid. The equilibrium outcome will be that the bidder with the highest valuation will win and pay the second highest valuation. Similar to the English auction, the second-price sealed-bid auction is efficient and truth-revealing.

D. SIGNIFICANT FACTORS FOR DETERMINING AUCTION FORMATS

Auction design affects revenue generation and bidders' behavior. In particular, it is important to minimize the risk of collusion and entry-deterrents. As highlighted by Homb (2006), there are several factors to consider when deciding on the format of an

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[7] Paul Milgrom, "Auctions and Bidding: A Primer" Journal of Economic Perspectives Vol. 3 No.3 (Summer 1989), 8.

auction. These include revenue generation, bidder risk tolerance, collusion, reserve price, private information, number of bidders and other levers as summarized in the following assuming a forward auction.

1.　　Revenue Generation

According to the Revenue-Equivalence Theorem, the English auction, the Dutch auction, the first-price sealed-bid auction, and the second-price sealed-bid auction each generates the same revenue on average assuming:

- Risk-neutrality

- Independent private values

- Symmetric bidders

- Payment is based only on bids

The English and second-price sealed-bid auctions effectively produce the same revenue because the winner pays the second highest valuation as described earlier. The Dutch and first-price sealed-bid auctions yield the same revenue given that they are identical setups. Under the Dutch and first-price sealed-bid auctions, each bidder will choose to bid as small a premium as possible over the second highest valuation to maximize his economic rent. If all bidders are risk-neutral and symmetric, the winning bid will be close to the second highest valuation.

Even under these assumptions, however, the four auction formats do not necessarily produce the same outcome in all situations (i.e. for all possible combinations of bidder valuations); they generate the same prices only on average (across any given prior distribution of valuations). In addition, the theorem does not hold when the assumptions are violated.

2.　　Bidder Risk Tolerance

The extent and impact of information asymmetry differs across the four auction forms. Bidders can observe their competitors' bids and infer their valuation as well as

bidding strategies in an open, ascending-bid auction like the English auction. While this information is not available in the second-price sealed-bid auction, it does not affect how bidders choose to bid. Conversely, in the Dutch and first-price sealed-bid auctions, participants bid not knowing how others are going to bid, which creates uncertainty and increases risk.

If bidders are risk-neutral, the increased risk does not affect their behavior under certain given conditions. However, if bidders are risk-averse, they will tend to bid more aggressively to increase the probability of winning and reduce the level of uncertainty, even though this will decrease their potential economic rent. Therefore, with risk-averse bidders, the Dutch and first-price sealed-bid auctions will generate a higher expected price than the English and second-price sealed-bid auctions.

3.   Collusion

Collusion allows participants to keep prices down at the expense of the seller. The choice of auction design affects the risk of collusion, be it overt or tacit among bidders. Unlike open auctions where participants can send signals through their bids, it is more difficult for bidders to communicate and collude in sealed-bid auctions. In addition, an auctioneer can take steps to counter collusion for example by setting a reserve price or excluding bidders who are known to be part of a collusion ring.

4.   Reserve Price

A reserve price, which is the minimum price for which a seller is willing to sell a good or service in a forward auction, or the maximum price a buyer is willing to buy a good or service in a reverse auction, can be specified to guarantee a minimum revenue or maximum cost. Reserve prices should be set carefully to ensure that they do not inadvertently discourage participants from bidding or result in the loss of profitable transactions.

Reserve prices can be used to decrease the gains from collusion and thereby reduce the incentive for bidders to co-operate with each other. The use of reserve prices

as a tool to prevent collusion is more common in auctions where participants can more easily collude, for example if the pool of potential bidders is small and they know each other.

5. Private Information

The auctioneer has to decide on whether and how much information should be disclosed to participants, which can affect their valuation. The information may provide bidders knowledge of the product or service, quantity available, historical sales data or the competition involved. Sellers are motivated to reveal information that influences bidders' valuations upwards and increases revenue. It is not in their interest to provide information that will cause bidders to revise their valuations downwards.

In addition, information can also affect uncertainty, which impacts participants' behavior. For example, a seller can reveal information to increase uncertainty in Dutch and first-price sealed-bid auctions with risk-averse bidders to induce them to bid more aggressively and thereby generate higher expected revenues.

6. Number of Bidders

In general, an increase in the number of bidders increases the amount of competition and the expected revenues in the Dutch and first-price sealed-bid auctions, because participants will tend to bid more aggressively to increase their chances of winning. Bidders do not, however, change the way they bid in the English and second-price sealed-bid auctions because their dominant strategy is to bid their valuation regardless of the number of competitors and what they bid.

Uncertainty regarding the number of bidders in an auction with risk-averse bidders increases the expected revenue generated in the Dutch and first-price sealed-bid auctions. There is no impact if bidders are risk neutral. As explained before, uncertainty does not affect bidding in the English and second-price sealed-bid auctions.

7. Other Levers

There are additional levers that can be used to enhance the auction design. First, a fixed and non-refundable entry fee can be charged for participation rights. An entry fee can serve to keep out undesirable participants such as those who are not serious bidders as well as function as a reserve price by excluding buyers with low valuations. However, care should be taken in setting the entry fee to ensure that it does not discourage serious participants and limit competition. An entry fee may not be suitable in an assignment setting because individuals are likely to be put off by the non-refundable expense with no guarantee of any return.

Second, a time limit for bid submission can be imposed to reduce bidders' chances of researching or meeting each other and determining others' valuation or bidding strategies. Setting a time limit increases uncertainty for bidders which can lead them to bid more aggressively. A time limit, however, may not be suitable in a military assignment auction because service members are dispersed all over the world and individuals may not receive information on or submit bids within a short timeframe.

Third, a middleman can be used to represent bidders who wish to remain anonymous or are unable to be present at the auction. Middlemen will need to know bidders' valuation and therefore maximum bid. It is also best to involve middlemen who have a good understanding of the transacted item. In the case of a service member who may not be able to submit a bid given his geographic location, he can still participate in the assignment auction through a middleman.

E. CHAPTER SUMMARY

This chapter provided an overview of auction theory by first describing key features of auctions; distinguishing between single-object and multi-object auctions, common value and independent private-values auctions, forward and reverse auctions, open and sealed-bid auctions, as well as first-price and second-price auctions. In addition, the four basic auction forms (the English auction, the Dutch auction, the first-price sealed-bid auction; and the second-price sealed-bid auction) were explained. Finally, several significant factors for determining auction formats were presented.

These factors were revenue generation, bidder risk tolerance, collusion, reserve price, private information, number of bidders and other levers.

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III. APPLICATION OF AUCTIONS FOR ASSIGNMENTS IN THE MILITARY

Auctions can be used to trade a wide variety of products and services including job assignments in the military. The U.S. Navy has implemented a bidding system to establish the pay incentive for sailors who volunteer for billets that are traditionally less desirable. There are several considerations in designing an auction in this context. In addition, complications arise with the use of pure auctions, which are better resolved by an alternative assignment mechanism.

A. AUCTION FORMAT CONSIDERATIONS FOR ASSIGNMENTS IN THE MILITARY

An assignment auction typically involves auctioning several items at a time. It is an independent private-values auction because service members' valuation of a post will differ based on personal taste and preference. Each service member's valuation of a particular billet is influenced by family circumstances and geographic bonds, educational qualifications, skills set and career plan among other factors. An assignment auction will also by necessity be a reverse auction: The military is the one buyer or bid-taker for manpower services and eligible service members are the many sellers (of labor services) who are the bidders. Service members compete on the pay incentive they require to accept a particular post and the right to be deployed is won by the member with the lowest bid.

Either an open or sealed-bid auction with the final pay incentive calculated based on the winning bid or the second lowest bid can be used to determine assignments. Homb (2006) compares the four basic auction forms using five decision criteria, which are efficiency, cost effectiveness, equity, practicality and manipulability. The highlights of the analysis are presented below.

1. Efficiency

Economic efficiency is achieved when the joint surplus of the bid-taker and the winning bidder is maximized. If the bid-taker (the military) is concerned only with minimizing the amount of the incentive pay and is indifferent about which bidder (service member) wins the auction, then the joint surplus associated with a particular assignment will be maximized whenever the bidder with the lowest willingness-to-accept (i.e. the service member willing to take the assignment for the least amount of incentive pay) wins the auction and receives the assignment. Note that the joint surplus is independent of the actual pay incentive paid.

English and second-price sealed-bid assignment auctions for an individual billet are theoretically Pareto-efficient, because the bidder with the lowest willingness-to-accept will always win the post, given that each bidder's dominant strategy in each auction is to bid his true valuation. Unlike these auctions, an efficient equilibrium outcome is not guaranteed in the Dutch and the first-price sealed-bid auctions unless all bidders have the same information (or expectations). However, under general conditions, the expected outcome is the same for all four auction forms on average, with each generating a winning bid approximately equal to the second lowest bid.

Despite the theoretical underpinnings of the second-price sealed-bid auction, research has shown that it may not be efficient because bidders often stray from the dominant strategy and submit bids over their true valuation in a forward auction, or under their true valuation in a reverse auction. This may be because the bidders are uninformed or do not understand the auction form and the resulting dominant strategy.

2. Cost Effectiveness

The cost effectiveness of the assignment auction is important to the military as it seeks to keep pay costs down. All else being equal, an auction that distributes the joint surplus most favorably to the employer will be preferred. Based on the Revenue-Equivalence Theorem, the English auction, the Dutch auction, the first-price sealed-bid auction, and the second-price sealed-bid auction will yield the same average surplus for the military.

The Revenue-Equivalence Theorem presupposes that bidders are risk-neutral, however. In the case where service members are risk averse, they will bid more aggressively in the Dutch and first-price sealed-bid auctions to increase their chances of winning, even though this decreases the pay incentive bids and transfers greater surplus to the employer. Furthermore, an increase in the number of bidders increases the amount of competition and the expected employer surplus in the Dutch and first-price sealed-bid auctions because participants will tend to bid more aggressively to increase their chances of winning. As such, the military will do better with the Dutch and first-price sealed-bid auctions if bidders are risk averse and there is a significant pool of bidders to minimize cost and maximize employer surplus.

3.   Equity

Equity, in terms of equal pay for individuals with the same characteristics (for example, educational qualifications, relevant experience and years of service) doing the same jobs, is generally not achievable in a simple assignment auction regardless of the auction form chosen. This is because the incentive pay for any individual is determined only by the valuations and competition within the particular auction that he/she wins, Therefore the incentive pay for two similar individuals can vary significantly based on varying valuations and levels of competition across assignments.

4.   Practicality

The open auctions are more difficult to implement in an assignment setting because all eligible service members have to be gathered or represented to conduct the auction. In contrast, the sealed-bid auctions are much easier to carry out because bidders only need to know the bid submission format and to submit their bids by the deadline specified.

Comparing the two sealed-bid auction forms, the first-price sealed-bid auction is more familiar to and easier to comprehend for a first-time user because the winner is the one who submitted the lowest bid and will simply receive the pay incentive he bid. The second-price sealed-bid auction is somewhat less familiar to most people. Furthermore,

the strategic behavior requirements are less intuitive so people will find it harder to understand without substantial education. On the other hand, if bidders <u>did</u> know the equilibrium bidding strategies under each auction format, implementing the second-price bidding strategy will be far simpler for the bidders.

5. Manipulability

An auction can be subject to manipulation by bidders who improve their personal results by not revealing their true valuations. For example, groups of bidders can collude or individual bidders can exhibit unexpected behavior. The risk of collusion is higher in the open auctions because bidders can observe competitors' behavior and send signals through the bidding process. This risk is reduced if the number of participants is large and bidders are not familiar with the auction format or each other.

Besides the choice of auction form, employers can set a reserve price equal to the maximum WTP to fill the post to avoid overpayment. Employers can also impose a time limit for bid submission and keep private information that may lead bidders to seek greater surplus value, such as the number of posts that the employer needs to fill.

A bid-taker can also manipulate auction results by cheating. For example, an employer can feign a second-lowest bid that is just above the lowest bid to lower salary costs in second-price sealed-bid auctions. Finally, if considerations other than relative bids are used to determine auction winners, there may be an opportunity for bidders to benefit from deception, as will be explained further in the next section.

B. COMPLICATIONS WITH USE OF AUCTIONS FOR ASSIGNMENTS IN THE MILITARY

Based on the above considerations, it will appear that first-price sealed-bid auctions are the most suitable for determining assignments in the military. Nevertheless, as observed by Homb (2006), there are some complications with using the first-price sealed-bid auction that affect the incentive of bidders to submit a truthful valuation of the jobs.

Firstly, an assignment auction is a multi-object auction and a service member can submit bids on more than one job. However, unlike multi-object auctions in which sellers (buyers) can sell (buy) more than one item, a service member can only win one post, even if he is the lowest bidder for more than one post. As such, service members may overstate their minimum WTA or reservation wage because they anticipate that the system will allocate them a post, if the number of open posts is at or above the number of bidders.

Secondly, auctions assume one-sided preferences (i.e., only bidders have preferences). The use of pure auctions implies that the value of the pay incentive bids is the only factor the employer cares about. However, employers are also concerned with the quality of the assignee. If a quality variable is included along with the bid value to reflect employer preferences over which employee gets which job, high quality bidders have an incentive to maximize their personal surplus by submitting a higher bid than their true valuation of the post.

C.   AN ALTERNATIVE AUCTION FORMAT FOR ASSIGNMENTS IN THE MILITARY

Given that there are complications with the use of a pure auction mechanism, Homb (2006) proposed an alternative auction mechanism that combined elements of both auction theory and matching theory[8] to overcome these complications and potentially reduce the U.S. Department of Defense (DoD)'s cost.

Under the proposed alternative auction mechanism, the bid-taker or employer is required to identify the reservation price (i.e., maximum WTP) for each post to be filled. Bidders or service members will be required to submit their pay incentive bid, which is their reservation wage or minimum WTA for each post. Each service member may have multiple preferred posts, which will be reflected in his bids.

Given this information, an attempt is made to match a post to each service member that maximizes his surplus, which is equal to the reservation price minus his

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[8] Matching theory focuses on the trading of multiple unique items between sellers and buyers both of whom have preferences over the outcomes.

reservation wage. Feasible matches of posts and service members are set temporarily. If there is an excess demand for posts, the reservation price is lowered and the matching process is repeated until a competitive equilibrium is reached with one-to-one matches of service members to their most preferred post under the current set of reservation prices. Service members are motivated to bid truthfully because the mechanism starts from each post's reservation price and only adjusts it downwards if there is excess demand.

To incorporate employer preferences, minimum qualification levels can be set for each post. Where to set the minimum qualification level for each billet is an important consideration for the bid-taker, as the mechanism will assign no additional value to service members who possess additional valuable qualifications above and beyond these minimum qualifications. Classifying each bidder as simply either "qualified" or "not qualified" to bid on a particular billet allows the auction mechanism to retain its truth-revealing feature by only comparing pay incentive bids to determine assignments.

This alternative auction mechanism can be interpreted as a generalization of the second-price sealed-bid auction. The assignments made and the final incentive pay levels calculated through this mechanism are such that no service member assigned to a billet will prefer to be assigned to a different billet (and receive the particular incentive pay associated with that billet) and, moreover, no service member who is NOT assigned to one of the billets eligible for such incentive pay will prefer to be assigned to one of those billets given the final incentive pay levels. In other words, given the final incentive pay levels calculated by the mechanism, each service member receives his most preferred assignment. Hence, the outcome of this mechanism is a "stable" matching between service members and billets.

An additional appealing characteristic of the mechanism is that it is truth-revealing for all service members. On the negative side, some posts may go unfilled as a result of this mechanism, however this is likely the result of the employer either setting the minimum qualifications too high or the reservation price too low. The employer may therefore need to review the minimum qualifications and/or reservation price for unmatched posts if they remain unfilled after several auctions.

While the mechanism induces truthful behavior from service members, there is no means of ensuring that the employer will likewise submit honest minimum qualifications or reservation prices. As such, the employer should establish a clear set of guidelines and be as transparent as possible to manage employee perceptions.

In addition, the employer should note that while this is the minimum-cost truth-revealing mechanism, it is not necessarily the cost-minimizing mechanism because the mechanism starts from the reservation price and only adjusts it downwards if there is excess demand. The reservation price and minimum qualification requirement therefore need to be set carefully to balance the probability of filling the post with the cost and quality of the assignment.

D.   CHAPTER SUMMARY

This chapter presented the considerations in designing an auction in the military assignment settings, which include efficiency, cost effectiveness, equity, practicality and manipulability. Based on these considerations, the first-price sealed-bid auction was found to be the most suitable for determining assignments in the military. However, there were two major complications in an assignment auction that affect the incentive of bidders to submit a truthful valuation of the jobs. Firstly, an assignment auction is a multi-object auction in which a service member can only win one post even if he is the lowest bidder for more than one post. Secondly, an assignment auction assumes that the employer is only concerned about the value of the pay incentive bids and not with the quality of the assignment, which is not the case.

An alternative auction mechanism that combines elements of both auction theory and matching theory to overcome these complications was then outlined. This mechanism is truth-revealing for all service members and produces an outcome similar to that under a second-price sealed-bid auction.

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IV. SIMULATION AND EVALUATION OF THE ALTERNATIVE AUCTION FORMAT

The alternative auction mechanism outlined in Chapter III can potentially reduce manpower costs for the U.S. Department of Defense (DoD) because it is the minimum-cost truth-revealing mechanism. However, the model has not been tested to see how it works in reality. The following discussion further defines the proposed alternative auction mechanism for the U.S. Navy and presents the simulation setup for testing the effectiveness of the mechanism. A separate mechanism, which is not truth-revealing for sailors but possibly more cost-effective, is also set up to help the Navy evaluate and trade-off the cost of overpaying sailors for the job against the cost of truth-revelation. The operational performance and efficiency measures used to evaluate the mechanisms are then defined.

A. MODEL MODIFICATION

The original alternative auction mechanism is based on the assumption that additional qualifications do not matter much if service members can meet the minimum qualification requirements, which is not necessarily true. If additional qualifications above-and-beyond this minimum standard significantly affect employer preferences, the model may fall unacceptably short of maximizing total employer surplus across all billets.

The employer surplus for a particular billet in the U.S. Navy can be defined as the value of having a sailor man the billet over and above the wage paid. Total employer surplus is the sum of employer surpluses across all billets. The value of having a sailor man the billet depends on the fitness or qualifications of the sailor. To incorporate employer preferences fully, the employer should be required to identify the maximum WTP for each billet and category of sailor that can fill the billet, starting from the category with the lowest acceptable qualifications.

B.   SIMULATION SETUP

The modified alternative auction mechanism is tested by simulating the U.S. Navy auction assignment process involving 10 AIP billets and 10 sailors. Billet values or maximum WTPs are randomly generated from a uniform distribution ranging from 0 to 2,000 and rounded to the nearest bid increment, which is set at 50. This range of random values is intended to encompass the range of actual historic monthly assignment incentive pay amounts, which have been observed to range from virtually nothing to as high as $1,700. Sailor preferences or minimum WTAs are similarly randomly generated from a uniform distribution of the same range and rounded to the nearest bid increment. The same range is used for sailor preferences and billet values to avoid inadvertent manipulation of the evaluation measures, such as billet and sailor surpluses that are affected by the absolute size of the WTPs and WTAs. The range starts from zero to allow for the possibility that a particular sailor brings no additional value (above his base compensation) to a particular billet.

Under this auction mechanism, each sailor is assigned a billet that maximizes his surplus given an existing set of billet AIP levels (which will differ for each sailor). The initial AIP level associated with any sailor/billet combination is equal to the billet's reservation price for that sailor. Competition for a particular billet will repeatedly lower its AIP levels, and the assignment process does not end until there is no longer excess demand for any billet. The solution generated this way is considered "sailor-optimal" because the mechanism starts from each billet's reservation price and only adjusts it downwards if there is excess demand; sailors are thereby motivated to submit truthful bids. However, the mechanism is not necessarily truth-revealing or cost-minimizing for the Navy.

A separate auction mechanism can be set up such that a greater share of the surpluses accrues to the Navy instead of the sailors. Under this "billet-optimal" model, each billet selects and is temporarily matched to the sailor providing it the highest surplus, which is equal to the reservation price minus the AIP level for each sailor, with the AIP level initially set equal to the reservation wage that the sailor specified for that billet. Competition among the billets for a particular sailor raises the AIP levels for that

26

sailor and the matching process continues until a stable equilibrium is reached in which each billet is matched to its most preferred sailor under the current set of AIP wages without displacing another billet.

Such a mechanism is truth-revealing for the Navy and possibly more cost-effective as well, because sailor reservation wages instead of billet reservation prices form the starting point for the AIP wages. However, there is no guarantee that sailors will submit truthful bids under this setup. Both auction mechanisms are simulated to help the Navy evaluate and trade-off the cost of overpaying sailors for the job against the cost of truth-revelation. The test algorithms for both models are presented below.

1. Sailor-Optimal Model

Under the sailor-optimal model, the test algorithm starts with Sailor One and determines his value to each billet (i.e., each billet's maximum WTP for Sailor One). Sailor One's surplus for each billet is then computed by subtracting his minimum WTA from the AIP level, which is initially set equal to the billet's WTP. The sailor is temporarily assigned to the billet that provides him the highest surplus and the AIP associated with that billet is adjusted downwards by one bid increment for all subsequent sailors to reflect that there is competing demand for the post.

After Sailor One has been temporarily assigned, the algorithm moves onto Sailor Two and similarly determines which billet offers him the greatest surplus. Sailor Two is temporarily assigned to that billet even if it happens to be the temporary assignment for Sailor One, who is "bumped" out of the billet and moved down the list after Sailor 10 for reassignment by the algorithm. The AIP level of the billet to which Sailor Two is assigned is then also lowered by one bid increment for all subsequent sailors.

The algorithm is repeated until all possible one-to-one matches of sailors to their most preferred post have been made and no displacement takes place under the existing set of AIP wages. This final set of matches represents the optimal equilibrium assignments for sailors because the mechanism starts the AIP wages at the billet WTPs and only adjusts them downwards if there is excess demand.

2. Billet-Optimal Model

Conversely, under the billet-optimal model, the test algorithm starts with Billet One and determines each sailor's minimum WTA for the billet. Billet One's surplus is then computed by subtracting each sailor's AIP wage (initially set equal to the sailor's minimum WTA) from the corresponding WTP. The sailor that provides the highest surplus is temporarily assigned to Billet One and the sailor's AIP wage is adjusted upwards by one bid increment for all subsequent billets to reflect that there is competing demand for the sailor.

After a sailor has been temporarily assigned to Billet One, the algorithm moves onto Billet Two and similarly determines which sailor offers the greatest surplus. The sailor offering the greatest surplus is temporarily assigned to Billet Two even if it happens to be the same sailor who was temporarily assigned to Billet One, which will then become vacant again and moved down the list after Billet 10 for reassignment by the algorithm. The AIP wage of the assigned sailor is then again raised by one bid increment for all subsequent billets.

The algorithm is repeated until all possible one-to-one matches of billets to their most preferred sailor have been made and no displacement takes place under the existing set of AIP wages. This final set of matches represents the optimal equilibrium assignments for billets because the mechanism starts the AIP wages at the sailor WTAs and only adjusts them upwards if there is excess demand.

C. EVALUATION MEASURES

The measures that will be used to evaluate the two auction mechanisms are grouped into five categories, which are overall system performance measures, sailor value measures, billet value measures, cost measures and quality measures.

1. Overall System Performance Measures

This category of evaluation measures is concerned with the overall operational performance as well as economic efficiency of the mechanism and includes the following:

a.   ***Trial Success***

        (Solution?)

This measure considers if a solution was found for the trial within 200 iterations of the algorithm. It is used to evaluate the mechanism's success in finding a solution.

b.   ***Number of Iterations***

        (Rounds)

This measure considers the number of iterations required to find a solution. It is used to evaluate the mechanism's speed in finding a solution.

c.   ***Percentage of Sailors Assigned***

        (% of Sailors Assigned)

This measure considers the percentage of sailors successfully assigned by the algorithm. It further defines the mechanism's success rate in finding a solution.

d.   ***Percentage of Billets Filled***

        (% of Billets Filled)

This measure considers the percentage of billets successfully filled by the algorithm. Similar to the *Percentage of Sailors Assigned* measure, it further defines the mechanism's success rate in finding a solution.

e.   ***Ratio of Average Total Surplus Attained to Upper Bound of Distribution Range for WTP Values***

        (Avg Total Surplus/UB)

This measure considers the average total surplus attained as a proportion of the upper bound of the distribution range for WTP values. It is used to evaluate the average total surplus attained under the mechanism normalized by the maximum possible WTP for any sailor. The average total surplus attained is equal to the average of the difference between the matched billet's original reservation price and the assigned sailor's original reservation wage.

f.   ***Ratio of Total Surplus Attained to Maximum Total Surplus Attainable***

(Total Surplus (Current Model)/ Total Surplus (LP))

This measure considers the total surplus attained under the mechanism as a proportion of the maximum total surplus attainable given the WTA and WTP values generated for the trial. It is used to evaluate the economic efficiency of the mechanism. The maximum total surplus attainable is calculated using a linear programming model, which is described in the Appendix, to determine the assignment of sailors to billets that maximizes total surplus.

2.   Sailor Value Measures

This category of measures assesses the value generated by the mechanism for sailors and includes the following:

a.   ***Ratio of Average Sailor Surplus to Average Total Surplus Attained***

(Avg Sailor Surplus/Avg Total Surplus)

This measure considers the average surplus generated for sailors as a proportion of the average total surplus attained. It is used to evaluate the average share of the total surplus available that accrues to the sailors. The average sailor surplus is equal to the average of the difference between the assigned sailor's AIP for the matched billet and his original reservation wage.

b.   ***Ratio of Average Sailor Surplus to Upper Bound of Distribution Range for WTP Values***

(Avg Sailor Surplus/UB)

This measure considers the average surplus generated for sailors as a proportion of the upper bound of the distribution range for WTP values. It is used to evaluate the average sailor surplus produced by the mechanism normalized by the maximum possible WTP for any sailor.

c. ***Ratio of Average Sailor Surplus to Average AIP***
(Avg Sailor Surplus/Avg AIP)

This measure considers the average surplus generated for sailors as a proportion of the average AIP. It is used to evaluate the average sailor surplus produced by the mechanism normalized by the average wage payment. It indicates how much of the AIP wage that sailors receive is surplus enjoyed by the sailor as opposed to opportunity cost incurred as a result of serving in a less appealing billet.

3. **Billet Value Measures**

This category of measures assesses the value generated by the mechanism for billets and includes the following:

a. ***Ratio of Average Billet Surplus to Average Total Surplus Attained***
(Avg Billet Surplus/Avg Total Surplus)

This measure considers the average surplus generated for billets as a proportion of the average total surplus attained. It is used to evaluate the share of the total surplus available that accrues to the billets. The average billet surplus is equal to the average of the difference between the matched billet's original reservation price and the assigned sailor's AIP.

b. ***Ratio of Average Billet Surplus to Upper Bound of Distribution Range for WTP Values***
(Avg Billet Surplus/UB)

This measure considers the average surplus generated for billets as a proportion of the upper bound of the distribution range for WTP values. It is used to evaluate the average billet surplus produced by the mechanism normalized by the maximum possible WTP for any sailor.

c. ***Ratio of Average Billet Surplus to Average WTP***
(Avg Billet Surplus/Avg WTP)

This measure considers the average surplus generated for billets as a proportion of the average WTP. It is used to evaluate the average billet surplus produced by the mechanism normalized by the average maximum payment billets are willing to

make. It indicates how much of the value that a sailor brings to a position is actually captured or enjoyed by the billet (or Navy) relative to how much of this value is offset by AIP wages paid for the sailor's services.

4. Cost Measures

This category of evaluation measures is concerned with the manpower costs incurred by the Navy and includes the following:

a. *Ratio of Average AIP to Upper Bound of Distribution Range for WTP Values*

(Avg AIP/UB)

This measure considers the average AIP cost as a proportion of the upper bound of the distribution range for WTP values. It is used to evaluate the average wage payment generated by the mechanism normalized by the maximum possible WTP for any sailor.

b. *Average Ratio of AIP to WTP*

(Avg (AIP/WTP))

This measure considers the average of the ratio of the assigned sailor's AIP to the matched billet's WTP. It is used to evaluate the average size of the wage payment as a proportion of the matched billet's WTP for the assigned sailor.

c. *Ratio of Average AIP under Mechanism to Average AIP under Least-cost Sailor Model*

(Avg AIP (Current Model)/Avg AIP (Billet-Optimal, all WTPs = UB))

The desire to focus solely on controlling costs can lead the Navy to minimize the wage bill by assigning the least-cost sailor to each billet. However, it is generally not possible to do so unless sailors' true reservation wages are known. Alternatively, the Navy can require billets to select the least-cost sailor as their preferred choice. The solution set for this scenario is analogous to the results of the billet-optimal model in which all billet WTPs equal the upper bound of the range = 2,000. Under this model, each billet is temporarily matched to the sailor with the lowest WTA. Competition for any sailor raises that sailor's reservation wages and ultimately the

32

lowest-cost sailor is assigned to each billet. The model will be truth-revealing even for sailors because billets are not concerned with the quality of the assignee given that billets have the same WTP for all sailors.

This measure considers the average AIP cost under the mechanism as a proportion of the average AIP cost under the least-cost sailor model. It is used to evaluate the average wage payment generated by the mechanism normalized by the average wage payment resulting when each billet adopts the least-cost sailor selection policy.

5.   Quality Measures

This category of evaluation measures is concerned with the quality of the assignees as measured by the WTP values and includes the following:

   a.   ***Ratio of Average WTP to Upper Bound of Distribution Range for WTP Values***

   (Avg WTP/UB)

This measure considers the average WTP as a proportion of the upper bound of the distribution range for WTP values. It is used to evaluate the average assignee quality generated by the mechanism normalized by the maximum quality possible.

   b.   ***Average Ratio of WTP to Max WTP***

   (Avg (WTP/Max WTP))

This measure considers the average of the ratio of the matched billet's WTP for the assignee to that billet's maximum WTP. It is used to evaluate the average level of the assignee quality as a proportion of the quality of the most qualified sailor.

   c.   ***Ratio of Average WTP under Mechanism to Average WTP under Most Qualified Sailor Model***

   (Avg WTP (Current Model)/Avg WTP (Sailor-Optimal Model, all WTAs = 0))

The U.S. Navy can theoretically be prepared to pay all costs to equip each billet with the sailor most qualified for the job. This scenario can be simulated using the sailor-optimal model in which all WTAs = 0. Under this model, each sailor is

temporarily assigned to the billet that values him most highly i.e. the billet for which he is most qualified. Competition for any billet lowers that billet's reservation prices and ultimately the sailor most qualified for the job is assigned to each billet.

This measure considers the average WTP under the mechanism as a proportion of the average WTP under the most qualified sailor model. It is used to evaluate the average assignee quality generated by the mechanism normalized by the average assignee quality attained under a policy of matching the most-qualified sailor to the job.

D.   CHAPTER SUMMARY

This chapter explained the modification made to the alternative auction mechanism and described the simulation set-up that involved developing the test algorithm for two models, the sailor-optimal model and the billet-optimal model, to help the Navy evaluate and trade-off the cost of overpaying sailors for the job against the cost of truth-revelation. The measures that will be used to evaluate the models were then presented, which include overall system performance measures, sailor value measures, billet value measures, cost measures and quality measures.

V. RESULTS AND EVALUATION

A pilot run of the simulation set-up described in the previous chapter generated 100 trials for a preliminary analysis. Based on the preliminary analysis, the mechanisms did not always result in the most efficient allocation. Further analysis and testing were conducted to determine the source of inefficiency. An additional 1,000 trials were generated for the full analysis. The simulation results are summarized below.

A. SIMULATION RESULTS

The simulation results for the 100 trials generated are as follows:

1. 100 Trials

a. Overall System Performance Measures

Table 1 presents the overall system performance measures for the two auction mechanisms, the Sailor-Optimal Model and the Billet-Optimal Model, averaged across the 100 trials generated.

Table 1.   Overall System Performance Measures (100 Trials).

Overall System Performance Measures	Models	
	Sailor-Optimal	Billet-Optimal
Solution?	100.00%	100.00%
	[-]	[-]
Rounds	37	32
	[2 , 109]	[4 , 108]
% of Sailors Assigned	95.50%	95.40%
	[90.00% , 100.00%]	[90.00% , 100.00%]
% of Billets Filled	95.50%	95.40%
	[90.00% , 100.00%]	[90.00% , 100.00%]
Avg Total Surplus / UB	55.86%	55.92%
	[41.39% , 71.11%]	[41.67% , 71.11%]
Total Surplus (Current Model) / Total Surplus (LP)	99.86%	99.90%
	[97.86% , 100.00%]	[97.14% , 100.00%]

* The minimum and maximum values observed are reflected in parentheses as [Min, Max].

Both models produced a feasible solution with a match rate of at least 90% for each trial[9]. The Billet-Optimal Model yielded a solution more quickly on average. The average match rate was about the same for the two models – the Billet-Optimal Model produced a 100% match rate in 54 of the 100 trials as compared to 55 trials under the Sailor-Optimal Model.

The same set of sailor assignments was produced in 71 trials. Of the remaining 29 trials, the same total surplus was generated in 10 trials. The Billet-Optimal Model yielded a higher total surplus in 11 trials, resulting in a marginally higher ratio of average total surplus attained to the upper bound of the distribution range for WTP values. The average total surplus attained in either model was about 56% of the maximum possible WTP for any sailor. Neither model generated an efficient solution in all 100 trials, but the Billet-Optimal Model came closer to doing so. The extent to which the solution deviated from the maximum total surplus attainable was small with both models, producing solutions that were at least 97% as efficient as those under the linear program. This inefficiency may be partly due to the choice of the bid increment and will be investigated in the next section.

b.   *Sailor Value Measures*

The sailor value measures are shown in Table 2. The Sailor-Optimal Model generated greater value for sailors as expected. Over 70% of the average total surplus went to sailors under the Sailor-Optimal Model, three times that under the Billet-Optimal Model. Under the Sailor-Optimal Model, the average sailor surplus constituted about 40% of the maximum possible WTP for any sailor as compared to less than 15% for the Billet-Optimal Model. The average sailor surplus comprised 65% and 37% of the average AIP wage under the Sailor-Optimal Model and Billet-Optimal Model respectively.

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[9] The match rate is sometimes less than 100% to allow for unmatched sailors and billets at the end of the process whereby none of the billets that are unmatched will have a WTP that is greater than the WTA of any of the unmatched sailors.

Table 2.   Sailor Value Measures (100 Trials).

Sailor Value Measures	Models	
	Sailor-Optimal	Billet-Optimal
Avg Sailor Surplus / Avg Total Surplus	73.39% [46.02% , 99.60%]	24.15% [4.02% , 56.67%]
Avg Sailor Surplus / UB	41.12% [22.50% , 62.50%]	13.39% [2.22% , 29.25%]
Avg Sailor Surplus / Avg AIP	64.55% [43.89% , 82.55%]	36.63% [9.57% , 66.10%]

* The minimum and maximum values observed are reflected in parentheses as [Min, Max].

c.   Billet Value Measures

The value measures for billets are given in Table 3. As expected, the Billet-Optimal Model generated greater value for billets. Over 75% of the average total surplus went to billets and the average billet surplus constituted about 43% of the maximum possible WTP for any sailor, almost three times that under the Sailor-Optimal Model. The average billet surplus comprised 55% of the average maximum payment billets were willing to make under the Billet-Optimal Model as compared to 19% under the Sailor-Optimal Model.

Table 3.   Billet Value Measures (100 Trials).

Billet Value Measures	Models	
	Sailor-Optimal	Billet-Optimal
Avg Billet Surplus / Avg Total Surplus	26.61% [0.40% , 53.98%]	75.85% [43.33% , 95.98%]
Avg Billet Surplus / UB	14.74% [0.25% , 33.61%]	42.53% [18.06% , 61.94%]
Avg Billet Surplus / Avg WTP	18.93% [0.30% , 40.33%]	54.60% [23.64% , 75.20%]

* The minimum and maximum values observed are reflected in parentheses as [Min, Max].

d.   Cost Measures

In addition to the billet surplus, the U.S. Navy will be concerned with the total cost of AIP wages, which is presented in Table 4. The average wage payment was significantly higher under the Sailor-Optimal Model, constituting over 60% of the maximum possible WTP for any sailor as compared to 35% under the Billet-Optimal

Model. The average size of the wage payment as a proportion of the matched billet's WTP for the assigned sailor was 45% and 81% under the Billet-Optimal Model and the Sailor-Optimal Model respectively.

As compared to the least-cost sailor model, the average AIP cost under the Billet-Optimal Model was about 165% higher[10] while that under the Sailor-Optimal Model was almost 300% higher. The cost of truth revelation on the part of service members ranged from 50 - 240%[11] of the average wage payment resulting when each billet adopts the least-cost sailor selection policy. The cost savings from not overpaying sailors will need to at least match this for the Sailor-Optimal Model to be as cost-effective as the Billet-Optimal Model.

Table 4.   Cost Measures (100 Trials).

Cost Measures	Models	
	Sailor-Optimal	Billet-Optimal
Avg AIP / UB	63.31%	35.30%
	[44.17% , 83.25%]	[17.50% , 58.33%]
Avg (AIP / WTP)	81.07%	45.40%
	[59.67% , 99.70%]	[24.80% , 76.36%]
Avg AIP (Current Model) / Avg AIP	296.06%	164.62%
(Billet-Optimal, all WTPs = UB)	[112.03% , 632.65%]	[58.65% , 391.03%]

 * The minimum and maximum values observed are reflected in parentheses as [Min, Max].

e.   Quality Measures

The quality measures are comparable for the two models, as shown in Table 5. The average assignee quality generated by both mechanisms was 78% of the maximum quality possible and 91% of the average assignee quality under the most

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[10] There were 12 cases where the Billet-Optimal Model produced a lower average AIP cost than the least-cost sailor model although two-thirds of these cases were within 92% of the average AIP cost generated under the least-cost sailor model. These cases show that even as billets try to minimize the wage bill by selecting the least-cost sailor, instances may arise where billets end up competing for the same sailors and raising the average AIP cost more than if each billet were to uniquely select the best sailor based on both cost and quality attributes.

[11] This range is derived by subtracting the minimum average AIP cost as compared to the least-cost sailor model for the billet-optimal model from that for the sailor-optimal model (112.03% - 58.65% ≈ 50%) and by subtracting the maximum average AIP cost as compared to the least-cost sailor model for the billet-optimal model from that for the sailor-optimal model (632.65% - 391.03% ≈ 240%). The sailor-optimal model is truth-revealing for all service members but incurs a higher average AIP cost.

qualified sailor model[12]. On average, billets were filled with well-qualified sailors above the 85th percentile among available sailors.

Table 5.   Quality Measures (100 Trials).

Quality Measures	Models	
	Sailor-Optimal	Billet-Optimal
Avg WTP / UB	78.05%	77.83%
	[64.25% , 89.17%]	[64.25% , 89.17%]
Avg (WTP / Max WTP)	85.76%	85.62%
	[71.70% , 96.65%]	[71.70% , 96.01%]
Avg WTP (Current Model) / Avg WTP	90.76%	90.50%
(Sailor-Optimal Model, all WTAs = 0)	[76.25% , 103.98%]	[76.25% , 103.98%]

   * The minimum and maximum values observed are reflected in parentheses as [Min, Max].

2.   Changing the Bid Increment

The outcome of the sailor-optimal and billet-optimal models may depend on the order in which sailors and billets are assigned. There may be ex-post renegotiation opportunities (which require both sides being better off). However, if the wages generated under these models are close to the equilibrium wages obtained under a more exact auction mechanism, the outcome of these models will be or come close to one where there are no ex-post renegotiation opportunities. Demange, Gale and Sotomayor (1986) examined two progressive auction mechanisms, an "exact auction mechanism" and an "approximate auction mechanism", and proved that the prices generated by the former are the minimum equilibrium prices (in a forward auction, such as our billet-optimal model) while those obtained under the latter will differ from the minimum equilibrium prices by at most the product of the bid increment and the maximum number of possible matches. As such, by making the bid increment sufficiently small, the wages obtained under the sailor-optimal and billet-optimal models will approach the equilibrium wages required to ensure there are no ex-post renegotiation opportunities.

The bid increment was set at 50 for the trials to be consistent with the current AIP bid amount as well as for computational ease. To investigate the impact of the bid

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[12] The average assignee quality under either mechanism was higher than that under the most qualified sailor model in some trials. This may reflect the fact that the most qualified sailor model is conditioned on a 100% match rate, which adds an additional problem constraint.

increment for the 20 trials in which either of the mechanisms failed to generate an efficient solution, the bid increment was lowered to 4 so that the product of the bid increment and the maximum number of possible assignments was less than 50 (4 x 10 = 40). An additional 800 iterations were added to ensure that a solution was obtained for both models.

Table 6 presents the results. With the bid increment lowered, an efficient solution was obtained for a quarter of the trials in which an inefficient solution was initially produced by the Sailor-Optimal Model. The total number of trials (out of 100) that were efficient with the lower bid increment was 92. The new average efficiency was 99.91%. An efficient solution was generated in only 10% of the trials in which an inefficient solution was initially produced by the Billet-Optimal Model. The total number of trials (out of 100) that were efficient with the lower bid increment was also 92 and the new average efficiency similarly was 99.91%. The total surplus was unchanged in 70% and 85% of the trials under the Sailor-Optimal Model and Billet-Optimal Model, respectively. The results show that the choice of the bid increment does affect the reflected economic efficiency of either mechanism; however neither mechanism can be considered an efficient algorithm per se.

Table 6.   Impact of Lowering the Bid Increment

Models	Total Surplus		
	Unchanged	Improved	Maximized
Sailor-Optimal	14 (70%)	1 (5%)	5 (25%)
Billet-Optimal	17 (85%)	1 (5%)	2 (10%)

* The percentage breakdown is reflected in parentheses.

3.   1,000 Trials

The simulation results for the additional 1,000 trials generated are as follows:

a.   *Overall System Performance Measures*

The overall system performance measures (excluding the economic efficiency measure) averaged across the 1,000 trials for both the sailor-optimal and billet-optimal models are shown in Table 7. A feasible solution was obtained 100% of the time

at about the same rate under both models. The match rate ranged from 70% - 100%[13] and the average match rate was about the same for both models. The average total surplus attained in either model was about 57% of the maximum possible WTP for any sailor.

Table 7.   Overall System Performance Measures (1,000 Trials).

Overall System Performance Measures	Models	
	Sailor-Optimal	Billet-Optimal
Solution?	100.00%	100.00%
	[-]	[-]
Rounds	38	37
	[3 , 132]	[1 , 140]
% of Sailors Assigned	95.78%	95.67%
	[70.00% , 100.00%]	[70.00% , 100.00%]
% of Billets Filled	95.78%	95.67%
	[70.00% , 100.00%]	[70.00% , 100.00%]
Avg Total Surplus/UB	56.58%	56.65%
	[38.61% , 78.33%]	[38.61% , 78.33%]

* The minimum and maximum values observed are reflected in parentheses as [Min, Max].

b.   Sailor Value Measures

Table 8 presents the value measures for sailors. Greater value was generated for sailors by the Sailor-Optimal Model, about three times that produced by the Billet-Optimal Model. Although almost 75% of the average total surplus went to sailors under the Sailor-Optimal Model, there were instances where sailors received a lower proportion of the surplus, as low as 28%. As a proportion of the maximum possible WTP for any sailor, the average sailor surplus came to 42% under the Sailor-Optimal Model as compared to 14% for the Billet-Optimal Model. The average sailor surplus constituted over 65% of the average wage payment under the Sailor-Optimal Model, 27 percentage points higher than under the Billet-Optimal Model.

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[13] The match rate is sometimes less than 100% to allow for unmatched sailors and billets at the end of the process whereby none of the billets that are unmatched will have a WTP that is greater than the WTA of any of the unmatched sailors.

Table 8.   Sailor Value Measures (1,000 Trials).

Sailor Value Measures	Models	
	Sailor-Optimal	Billet-Optimal
Avg Sailor Surplus / Avg Total Surplus	74.29% [27.67% , 98.76%]	25.61% [0% , 66.48%]
Avg Sailor Surplus / UB	42.13% [15.83% , 67.22%]	14.42% [0% , 41.39%]
Avg Sailor Surplus / Avg AIP	65.54% [34.75% , 88.97%]	38.32% [0% , 78.83%]

* The minimum and maximum values observed are reflected in parentheses as [Min, Max].

c.   *Billet Value Measures*

The value measures for billets are provided in Table 9. The Billet-Optimal Model generated greater value for billets with just under 75% of the average total surplus going to billets as compared to 26% under the Sailor-Optimal Model. Under the Billet-Optimal Model, the average billet surplus was over 40% of the maximum possible WTP for any sailor and over half of the average actual WTP for any sailor, almost three times the value of the measures under the Sailor-Optimal Model.

Table 9.   Billet Value Measures (1,000 Trials).

Billet Value Measures	Models	
	Sailor-Optimal	Billet-Optimal
Avg Billet Surplus / Avg Total Surplus	25.71% [1.24% , 72.33%]	74.39% [33.52% , 100.00%]
Avg Billet Surplus / UB	14.45% [0.75% , 44.69%]	42.23% [16.94% , 68.75%]
Avg Billet Surplus / Avg WTP	18.51% [0.89% , 54.58%]	53.80% [23.19% , 81.31%]

* The minimum and maximum values observed are reflected in parentheses as [Min, Max].

d.   *Cost Measures*

The cost measures for both models are given in Table 10. Under the Sailor-Optimal Model, the average wage payment constituted 64% of the maximum possible WTP for any sailor, 28 percentage points higher than that under the Billet-Optimal Model. The average size of the wage payment as a proportion of the matched billet's WTP for the assigned sailor was 46% and 81% under the Billet-Optimal Model and the Sailor-Optimal Model, respectively. The average AIP cost as compared to the

least-cost sailor model is more than 300% higher under the Sailor-Optimal Model and about 170% higher under the Billet-Optimal Model.

Table 10.   Cost Measures (1,000 Trials).

Cost Measures	Models	
	Sailor-Optimal	Billet-Optimal
Avg AIP / UB	63.95%	36.23%
	[35.83% , 86.25%]	[14.25% , 60.75%]
Avg (AIP / WTP)	81.49%	46.20%
	[45.42% , 99.11%]	[18.69% , 76.81%]
Avg AIP (Current Model) / Avg AIP	309.24%	172.61%
(Billet-Optimal, all WTPs = UB)	[114.94% , 1345.83%]	[48.28% , 576.47%]

* The minimum and maximum values observed are reflected in parentheses as [Min, Max].

e.   Quality Measures

Table 11 shows the quality measures. The average assignee quality under either mechanism was 78% of the maximum quality possible and 91% of the average assignee quality under the most qualified sailor model. Billets were filled with well-qualified sailors ranking above the 85th percentile among available sailors on average for both models. This similarity in terms of quality may be due to the fact that both models assign sailors to billets based on the total surplus possible (adjusting for competition) and high total surplus for a match requires both high WTP (quality) and low WTA.

Table 11.   Quality Measures (1,000 Trials).

Quality Measures	Models	
	Sailor-Optimal	Billet-Optimal
Avg WTP / UB	78.40%	78.46%
	[57.75% , 92.81%]	[59.25% , 92.81%]
Avg (WTP / Max WTP)	85.74%	85.81%
	[65.32% , 99.35%]	[65.17%, 99.35%]
Avg WTP (Current Model) / Avg WTP	90.59%	90.66%
(Sailor-Optimal Model, all WTAs = 0)	[69.54% , 107.80%]	[70.75% , 107.80%]

* The minimum and maximum values observed are reflected in parentheses as [Min, Max].

B.   EVALUATION OF THE ALTERNATIVE AUCTION MECHANISM

The results generated by the Sailor-Optimal Model were comparable to those under the Billet-Optimal Model where overall system performance and quality are

concerned. Both mechanisms are flexible enough to allow for unmatched sailors and billets at the end of the process whereby none of the billets that are unmatched will have a WTP that is greater than the WTA of any of the unmatched sailors. In other words, no mutually beneficial match can take place among the sailors and billets that are left over. This is in line with the philosophy of the AIP program which is not to force sailors unwillingly into these unappealing billets, and the billets certainly will not want a sailor who has more cost than value. Furthermore, each sailor and billet is eligible for several assignment iterations, so a small number of unmatched sailors and billets in one iteration may not be a significant issue. A dynamic model can examine how many iterations it will take to ensure an acceptable probability that any particular sailor or billet is matched, but that is for future research. Neither mechanism can be considered an efficient algorithm per se, although the extent to which the obtained solution deviated from the optimal solution was small; both models produced solutions that were over 99% efficient on average.

As can be expected, the Sailor-Optimal Model works to the advantage of sailors while the Billet-Optimal Model works in favor of the billets. The Sailor-Optimal Model (Billet-Optimal Model) generated a value up to three times greater than that under the alternative model for sailors (billets). The average sailor surplus constituted about two-thirds of the average wage payment under the Sailor-Optimal Model, 1.7 times that under the Billet-Optimal Model. Billets were able to save over half of the average maximum payment they were willing to make under the Billet-Optimal Model, as compared to under a fifth with the Sailor-Optimal Model. In terms of cost, the average wage payment was more than 1.7 times higher under the Sailor-Optimal Model, as compared to the Billet-Optimal Model, regardless of how it is normalized.

The preceding analysis is premised on the submission of truthful valuations by sailors and billets. Under the Sailor-Optimal Model, the initial AIP levels for a billet are set equal to the WTP of that billet for each sailor. These AIP levels are only "bid down" when there are multiple sailors who find that particular billet to be the most attractive assignment at any particular stage of the mechanism. Thus, under the Sailor-Optimal model, sailors have no incentive to misrepresent their true WTA for each billet, because submitting a higher than honest WTA (in a misguided attempt to raise the AIP wage),

will have no effect on the initial AIP levels and can only cause the mechanism to assign the deceptive sailor to a billet that is not his best option. A sailor who deceptively raises his submitted WTA in the Sailor-Optimal model can, in fact, reduce competition for a billet, and thus indeed produce a higher AIP level for that billet; however this is only achieved by taking this deceptive sailor out of competition for that billet. In other words, a sailor can use deception to produce higher AIP levels for a billet (or billets), but can only do so for billets to which he will not be assigned (while he is instead assigned to a less attractive billet). Thus, it is always in a sailor's best interest to truthfully report his WTA for each billet in the Sailor-Optimal model.

On the other hand, it is possible for the Sailor-Optimal model to <u>not</u> be truth-revealing for billets. This can only occur, however, if a billet knows in advance the minimum AIP wage that different sailors are willing-to-accept to be assigned to that billet. If a billet had such information about sailor WTAs (and also knew where its own WTP for sailors stood relative to other billet WTPs), there are scenarios in which a billet can theoretically increase its own surplus by lowering its bid for a sailor or sailors who were known to prefer that billet. This sort of "manipulation" by billets in the Sailor-Optimal model presents little worry, however, because it seems quite unlikely that a billet will have such good information about sailor WTAs (as well as about the WTPs of other billets).

By a symmetric argument, it can be shown that the Billet-Optimal Model, conversely, is truth-revealing for billets but possibly not for sailors, given that the mechanism is set up to favor the billets. A sailor who knows that he was a billet's only or preferred choice and that his WTA for that billet was also relatively low can conceivably maximize his personal surplus by submitting a higher bid than his true valuation of the post.

Comparing the two models in terms of truth revelation, it can be argued that it is more likely for sailors to know billet preferences than for billets to accurately guess sailor preferences, especially if the number of sailors is large and each sailor is only interested

in a limited number of billets. As such, the Sailor-Optimal Model may be more incentive-compatible overall and thereby more truth-revealing than the Billet-Optimal Model.

Given that the Sailor-Optimal Model is more costly but more truth-revealing overall than the Billet-Optimal Model, the Navy will need to weigh the potential cost savings from not overpaying sailors against the cost of truth revelation, and determine if the Sailor-Optimal Model is more cost-effective. This requires an empirical study or laboratory experimentation to determine if and how much sailors might game a billet-optimal mechanism.

C.   CHAPTER SUMMARY

This chapter reported the simulation results for a preliminary 100 trials, which encompassed overall system performance, sailor value, billet value, cost and quality measures. It was observed that neither the Sailor-Optimal Model nor the Billet-Optimal Model generated an efficient solution all the time. Further analysis and testing were conducted by changing the bid increment. Results showed that neither mechanism can be considered an efficient algorithm per se. The full analysis, based on an additional 1,000 trials was then presented. Finally, the proposed alternative auction mechanism was evaluated for these 1000 trials.

VI. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY

This research focused on an alternative auction mechanism proposed by Homb (2006), which combined elements of both auction theory and matching, to overcome complications in an assignment auction that affect the incentive of bidders to submit a truthful valuation of the jobs. The mechanism was further defined to fully incorporate employer preferences by requiring each billet to specify the maximum WTP for each category of sailor that can fill the billet, starting from the category with the lowest acceptable qualifications. This helps reduce the incentive for sailors to maximize their personal surplus by submitting a higher bid than their true valuation.

The modified alternative auction mechanism, the Sailor-Optimal Model, was tested to see if it works in reality by simulating the U.S. Navy auction assignment process involving 10 AIP billets and 10 sailors. A separate mechanism, the Billet-Optimal Model, which is not truth-revealing for sailors but possibly more cost-effective, was also set up to help the Navy evaluate and trade-off the cost of overpaying sailors for the job against the cost of truth-revelation. The simulation setup involved developing test algorithms for both models. Five categories of measures were proposed to evaluate the two mechanisms. These include overall system performance, sailor value, billet value, cost and quality measures.

100 trials were generated for a preliminary analysis. Results showed that the two models did not always generate the same set of sailor assignments or the same total surplus. However, they were close in terms of total surplus and were almost 100% efficient. Further analysis and testing indicated that part of the inefficiency was due to the choice of the bid increment; however, neither mechanism is a 100% efficient algorithm even after adjusting for the bid increment.

For the full analysis, an additional 1,000 trials were generated. In terms of overall system performance and quality, the Sailor-Optimal Model was found to be comparable to the Billet-Optimal Model. A feasible solution was obtained 100% of the time and relatively quickly. The match rate ranged from 70% - 100%; the average match rate was

over 95%. The average total surplus attained in either model was about 57% of the maximum possible WTP for any sailor. On average, assignee quality was 78% of the maximum quality possible and 91% of the assignee quality attained under a policy of matching the most-qualified sailor to the job. Both models filled billets with well-qualified sailors, above the 85th percentile.

Due to the setup of the models, the Sailor-Optimal Model produced sailor value up to three times greater than the Billet-Optimal Model, and vice versa. The average sailor surplus constituted about two-thirds of the average AIP wage under the Sailor-Optimal Model, 1.7 times that under the Billet-Optimal Model. Billets had to pay sailors, on average, less than half of their value to the billet (WTP) under the Billet-Optimal Model, but more than 80% of their value under the Sailor-Optimal Model. The Sailor-Optimal Model is more costly for the Navy with the average wage payment over 1.7 times higher as compared to the Billet-Optimal Model, regardless of how it is normalized.

Although the Sailor-Optimal Model is more costly, it is also likely to be more incentive-compatible and thereby more truth-revealing for both sailors and billets than the Billet-Optimal Model. This reflects that it is more likely for sailors to know billet preferences than for billets to accurately guess sailor preferences.

B.   CONCLUSIONS

The modified alternative auction mechanism was found to be viable; it produced a feasible solution consistently and easily. Although it was not efficient per se, the extent of economic inefficiency is small and the mechanism did generate an equilibrium solution in which sailors all prefer their final assignment over all others while billets are basically indifferent among all sailors given the final assignments and AIP levels. The mechanism worked well, generating an average match rate of over 95% and filling billets with well-qualified sailors above the 85th percentile on average. Billets retained, on average, almost a fifth of the value that the assigned sailor brought to the post. Under the mechanism, the average sailor surplus constituted about two-thirds of the average wage payment. The mechanism was relatively more costly for the Navy as compared to one set

up to shift surpluses to the Navy instead of the sailors, but it holds the advantage of being overall more incentive-compatible and thereby more truth-revealing.

C. RECOMMENDATIONS FOR FUTURE RESEARCH

There are several areas of follow-up from this research. First, an empirical study using laboratory experiments can be conducted to determine if sailors and billets understand how to use the mechanism as well as whether and to what extent they will engage in gaming or deception. Second, ex-post bargaining can be incorporated into the model to analyze the economic efficiency of the mechanism. Third, the mechanism can be further tested using alternative settings by varying the number of sailors and billets and the valuation (WTA/WTP) ranges. Fourth, only varying billet WTP levels across groupings or categories of sailors, based on rank and other standardized qualifications, rather than varying WTP across all individual sailors, will perhaps provide a better simulation of how billets will actually report their WTP levels if this mechanism were to be implemented. Finally, perhaps additional modifications to the mechanism can be explored that will retain the truth-revealing advantage of the sailor-optimal model while incorporating some of the additional cost savings of the billet-optimal model.

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APPENDIX. LINEAR PROGRAMMING MODEL

The setup of the linear programming model to determine the assignment of sailors to billets that maximizes total surplus is presented below.

The decision variables are the assignments of sailors to billets, X_{ij}. $X_{ij} = 1$ when sailor i is assigned to billet j and $X_{ij} = 0$ when sailor i is not assigned to billet j. The objective function is to maximize the total surplus, which is the sum of the surplus values for sailor-billet matches, where the surplus value is defined as the difference between the matched billet's WTP for the assigned sailor and that sailor's WTA for that billet. Besides the non-negativity constraint, the other constraints are that no sailor can be assigned to more than one billet and no billet can be matched to more than one sailor.

The mathematical formulation of the linear programming model is as follows:

$$\text{Max}_{X_{i,j}} \sum_{i,j} (WTP_{i,j} - WTA_{i,j}) \bullet X_{i,j}$$

subject to

$$\sum_{j} X_{i,j} \leq 1 \qquad \text{for } i = 1, 2, \ldots, 10$$

$$\sum_{i} X_{i,j} \leq 1 \qquad \text{for } j = 1, 2, \ldots, 10$$

$$X_{i,j} \geq 0 \qquad \text{for all } i, j$$

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